

ON THE MARKETING RESEARCH OF CONSUMER PRICES AND INFLATION PROCESS

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ABSTRACT. *Marketing information is used by financial and insurance institutions, business enterprises and companies for planning, control, monitoring and forecasting purposes in business. One of the problems is the detection and investigation of factors, which influence the behavior of consumers. Such a basic factor is, for instance, the consumer prices index. The significance of this factor periodically changes and depends on the values of main indexes of economy such as export, import, taxes, labor force, unemployment, inflation level, etc., and also on the behavior of consumers, their taste, living standard and style. For the marketing research of this dependence it is necessary to construct mathematical models of the evolution of consumer prices. In the paper, a new auto regression model with disturbances is constructed for consumer prices. The model includes monetary aggregate amount and control function. A new formula is derived for the solution of an equation for the consumer prices index, which can be used in forecasting the inflation process. Using the data on the consumer prices index in Georgia, a numerical example is given, which illustrates the estimates of the coefficients of the constructed model and the inflation process forecast.*

KEYWORDS: Marketing, consumer prices, inflation, auto regression model.

INTRODUCTION

The inflation process control is highly important for the stability of the consumer market. It is believed that inflation is the main destabilizing factor of market economy. The term “inflation” was used for the first time in Northern America during the civil war, and later came into use in Great Britain and France. In the economical literature, the notion of inflation became popular after World War I. The 20th century is called the epoch of inflation. The inflation process is typical of all countries. Hence it is important to have mathematical models adequately describing the character of inflation changes. The new auto-regression model is constructed in the paper and the explicit formula is derived for the consumer prices index. The formula depends on the time parameter and can be used for forecasting the inflation process.

A great number of monographs, text-books and scientific papers are dedicated to marketing research [1-3]. In these works essential use is made of statistical methods of investigation and, in particular, the method of regression analysis. In this context, a special reference should be made to the monograph [4] and the papers [5-7].

1. Let us consider the following auto regression model

$$p(k) = a_0 + a_1 p(k-1) + a_2 p(k-2) + \gamma m(k-1) + \beta \varepsilon(k), \quad (1)$$

where $p(k)$ is the consumer prices index at the moment of time k , $m(k)$ is monetary aggregate amount at the moment of time k , $\varepsilon(k)$ is a random component (random variable)

with zero mean. It can be produced by various disturbing factors, for instance, by the irregularity of import, export or investment flows, the economy instability, consumers' behavior and other factors. The values a_0 , a_1 , a_2 , γ and β are the numerical coefficients which are calculated by statistical data on $p(k)$.

We define the monetary aggregate as follows:

$$m(k) = \bar{m} + u(k), \quad (2)$$

where \bar{m} is the average value of the monetary aggregate, $u(k)$ is monetary aggregate increment. The function $u(k)$ is used as control. From (1), (2) we obtain the following equation for $p(k)$

$$p(k+2) - a_1 p(k+1) - a_2 p(k) = a'_0 + \gamma u(k+1) + \beta \varepsilon(k+2), \quad (3)$$

where $a'_0 = a_0 + \bar{m}$.

Let us consider the homogeneous equation

$$p(k+2) - a_1 p(k+1) - a_2 p(k) = 0, \quad (4)$$

whose solution has the form

$$p_0(k) = c_1 r_1^k + c_2 r_2^k,$$

where c_1 and c_2 are constants and the values

$$r_1 = \frac{a_1 + \sqrt{a_1^2 + 4a_2}}{2}, \quad r_2 = \frac{a_1 - \sqrt{a_1^2 + 4a_2}}{2}.$$

A particular solution of equation (3) will be sought in the form

$$p_1(k) = \mu_1(k) r_1^k + \mu_2(k) r_2^k. \quad (5)$$

Let

$$r_1^{k+1} \Delta \mu_1(k) + r_2^{k+1} \Delta \mu_2(k) = 0, \quad (6)$$

where

$$\Delta \mu_i(k) = \mu_i(k+1) - \mu_i(k), \quad i=1,2.$$

Substituting (6) into equation (3) we obtain

$$\begin{aligned} & (r_1^{k+2} \Delta \mu_1(k) + r_2^{k+2} \Delta \mu_2(k)) + (r_1^{k+2} \Delta \mu_1(k+1) + r_2^{k+2} \Delta \mu_2(k+1)) - \\ & - a_1 (r_1^{k+1} \Delta \mu_1(k) + r_2^{k+1} \Delta \mu_2(k)) + \\ & \mu_1(k) (r_1^{k+2} - a_1 r_1^{k+1} - a_2 r_1^k) + \mu_2(k) (r_2^{k+2} - a_1 r_2^{k+1} - a_2 r_2^k) = \\ & = a'_0 + \gamma u(k+1) + \beta \varepsilon(k+2). \\ & r_1^{k+2} \Delta \mu_1(k) + r_2^{k+2} \Delta \mu_2(k) = a'_0 + \gamma u(k+1) + \beta \varepsilon(k+2). \end{aligned} \quad (7)$$

Equations (6) and (7) imply

$$\Delta\mu_1(k) = -\frac{a'_0 + \gamma u(k+1) + \beta\varepsilon(k+2)}{(r_2 - r_1)r_1^{k+1}},$$

$$\Delta\mu_2(k) = \frac{a'_0 + \gamma u(k+1) + \beta\varepsilon(k+2)}{(r_2 - r_1)r_2^{k+1}},$$

$$\mu_1(k) = -\frac{1}{r_2 - r_1} \sum_{n=1}^k \frac{a'_0 + \gamma u(n) + \beta\varepsilon(n+1)}{r_1^k},$$

$$\mu_2(k) = \frac{1}{r_2 - r_1} \sum_{n=1}^k \frac{a'_0 + \gamma u(n) + \beta\varepsilon(n+1)}{r_2^k}.$$

Thus a particular solution of equation (3) has the form

$$p_1(k) = \frac{a'_0}{1 - a_1 - a_2} + \frac{\gamma}{r_2 - r_1} \sum_{n=1}^{k-1} (r_2^n - r_1^n) u(k-n) +$$

$$+ \frac{\beta}{r_2 - r_1} \sum_{n=1}^{k-1} (r_2^n - r_1^n) \varepsilon(k-n+1).$$

and a general solution of (3) is written as follows:

$$p(k) = c_1 r_1^k + c_2 r_2^k + \frac{a'_0}{1 - a_1 - a_2} + \frac{\gamma}{r_2 - r_1} \sum_{n=1}^{k-1} (r_2^n - r_1^n) u(k-n) +$$

$$+ \frac{\beta}{r_2 - r_1} \sum_{n=1}^{k-1} (r_2^n - r_1^n) \varepsilon(k-n+1),$$

Using the initial conditions $p(0)$ and $p(1)$, the constants c_1 and c_2 are defined by the relations:

$$c_1 = \frac{r_2}{r_2 - r_1} \left[p(0) - \frac{a'_0}{1 - a_1 - a_2} \right] - \frac{1}{r_2 - r_1} \left[p(1) - \frac{a'_0}{1 - a_1 - a_2} \right],$$

$$c_2 = \frac{1}{r_2 - r_1} \left[p(1) - \frac{a'_0}{1 - a_1 - a_2} \right] - \frac{r_1}{r_2 - r_1} \left[p(0) - \frac{a'_0}{1 - a_1 - a_2} \right].$$

Thus a general solution of equation (3) has the form:

$$p(k) = \frac{a'_0}{1 - a_1 - a_2} + \frac{\gamma}{r_2 - r_1} \sum_{n=1}^{k-1} (r_2^n - r_1^n) u(k-n) + \frac{\beta}{r_2 - r_1} \sum_{n=1}^{k-1} (r_2^n - r_1^n) \varepsilon(k-n+1) +$$

$$+ \frac{r_1 r_2 (r_2^{k-1} - r_1^{k-1})}{r_2 - r_1} \left[p(0) - \frac{a'_0}{1 - a_1 - a_2} \right] + \frac{r_2^k - r_1^k}{r_2 - r_1} \left[p(1) - \frac{a'_0}{1 - a_1 - a_2} \right]. \quad (8)$$

The obtained formula (8) is used for forecasting inflation. Let us write this formula for the time unit s . We have:

$$p(k+s) =$$

$$= \frac{a'_0}{1 - a_1 - a_2} + \frac{\gamma}{r_2 - r_1} \sum_{n=1}^{k-1} (r_2^n - r_1^n) u(k+s-n) + \frac{\beta}{r_2 - r_1} \sum_{n=1}^{k-1} (r_2^n - r_1^n) \varepsilon(k+s-n+1) +$$

$$+ \frac{r_1 r_2 (r_2^{s-1} - r_1^{s-1})}{r_2 - r_1} \left[p(k-1) - \frac{a'_0}{1 - a_1 - a_2} \right] + \frac{r_2^s - r_1^s}{r_2 - r_1} \left[p(k) - \frac{a'_0}{1 - a_1 - a_2} \right], \quad (9)$$

where $p(k-1)$ and $p(k)$ are the initial conditions at the moment of time k . Therefore a forecast function for step s has the form:

$$\hat{p}(k+s) = E_k [p(k+s)] = \frac{a'_0}{1-a_1-a_2} + \frac{\gamma}{r_2-r_1} \sum_{n=1}^s (r_2^n - r_1^n) u(k+s-n) + \frac{r_1 r_2 (r_2^{s-1} - r_1^{s-1})}{r_2 - r_1} \left[p(k-1) - \frac{a'_0}{1-a_1-a_2} \right] + \frac{r_2^s - r_1^s}{r_2 - r_1} \left[p(k) - \frac{a'_0}{1-a_1-a_2} \right]. \quad (10)$$

In particular, if $s=1$, then from (10) we obtain:

$$\hat{p}(k+1) = \hat{p}(k) + \gamma u(k),$$

if $s=2$, then

$$\hat{p}(k+2) = \frac{a'_0}{1-a_1-a_2} + \frac{\gamma}{r_2-r_1} \left\{ (r_2 - r_1) u(k+1) + (r_2^2 - r_1^2) u(k) \right\} + \frac{r_1 r_2^2 - r_2 r_1^2}{r_2 - r_1} \left[p(k-1) - \frac{a'_0}{1-a_1-a_2} \right] + (r_2 + r_1) \left[p(k) - \frac{a'_0}{1-a_1-a_2} \right],$$

and if $n=3$, then

$$\hat{p}(k+3) = \frac{a'_0}{1-a_1-a_2} + \frac{\gamma}{r_2-r_1} \left\{ (r_2 - r_1) u(k+2) + (r_2^2 - r_1^2) u(k+1) + (r_2^3 - r_1^3) u(k) \right\} + r_1 r_2 (r_1 + r_2) \left[p(k-1) - \frac{a'_0}{1-a_1-a_2} \right] + (r_1^2 + r_1 r_2 + r_2^2) \left[p(k) - \frac{a'_0}{1-a_1-a_2} \right].$$

2. Let us consider the following discrete auto regression model in the space of states

$$x(k+1) = A(k)x(k) + C(k)w(k), \quad (11)$$

$$z(k) = B(k)x(k) + v(k), \quad (12)$$

where $x(k)$ is an n -dimensional vector of states, $A(k)$ is an $n \times n$ matrix, $z(k)$ is a r -dimensional vector, $B(k)$ is a $r \times n$ matrix, $C(k)$ is an $n \times n$ perturbation matrix, $k=0,1,\dots$; $w(k)$ and $v(k)$ are respectively $n \times 1$ and $r \times 1$ non-correlated matrices.

Let the following conditions be fulfilled:

$$E\{w(k)v^T(j)\} = 0, \quad E\{w(k)\} = 0, \quad W\{v(k)\} = 0, \quad (13)$$

$$E\{w(k)w^T(j)\} = \begin{cases} Q(k), & k=j \\ 0, & k \neq j \end{cases}, \quad (14)$$

$$E\{v(k)v^T(j)\} = \begin{cases} R(k), & k=j \\ 0, & k \neq j \end{cases}, \quad (15)$$

where w^T and v^T denote the transposed vectors.

To investigate model (11), we will consider the following procedures.

Let us consider the auto regression model of order p

$$y(k+1) = a_0 + a_1 y(k) + a_2 y(k-1) + \dots + a_p y(k-p+1) + \varepsilon(k+1), \quad (16)$$

Let \bar{y} be the mean value of the process $y(k)$. Then (16) can be written in the form

$$y(k+1) - \bar{y} = a_1 [y(k) - \bar{y}] + a_2 [y(k-1) - \bar{y}] + \dots + a_p [y(k-p+1) - \bar{y}] + \varepsilon(k+1). \quad (17)$$

Let us write equation (17) for states and dimensions. We have

$$\begin{bmatrix} y(k+1) - \bar{y} \\ y(k) - \bar{y} \\ y(k-1) - \bar{y} \\ \vdots \\ y(k-p+2) - \bar{y} \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & \dots & a_{p-1} & a_p \\ 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \dots & 1 & 0 \end{bmatrix} \begin{bmatrix} y(k) - \bar{y} \\ y(k-1) - \bar{y} \\ y(k-2) - \bar{y} \\ \vdots \\ y(k-p+1) - \bar{y} \end{bmatrix} + \begin{bmatrix} \varepsilon(k+1) \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

$$z(k) = y(k) - \bar{y} + [1 \ 0 \ \dots \ 0] \begin{bmatrix} y(k+1) - \bar{y} \\ y(k) - \bar{y} \\ y(k-1) - \bar{y} \\ \vdots \\ y(k-p+2) - \bar{y} \end{bmatrix}.$$

Denote

$$x(k) = \begin{bmatrix} y(k+1) - \bar{y} \\ y(k) - \bar{y} \\ y(k-1) - \bar{y} \\ \vdots \\ y(k-p+2) - \bar{y} \end{bmatrix}, \quad \Phi = \begin{bmatrix} a_1 & a_2 & \dots & a_{p-1} & a_p \\ 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \dots & 1 & 0 \end{bmatrix},$$

$$w(k+1) = \begin{bmatrix} \varepsilon(k+1) \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

$$z(k) = y(k) - \bar{y}, \quad H = [1 \ 0 \ \dots \ 0].$$

Using these notations, we write (17) in the form

$$x(k+1) = \Phi x(k) + w(k+1),$$

$$z(k) = Hx(k).$$

Let us consider the auto regression process with a moving mean of arbitrary $n = \max\{p, q+1\}$ -dimension

$$y(k+1) = a_0 + a_1 y(k) + a_2 y(k-1) + \dots + a_n y(k-n+1) + \varepsilon(k+1) + \beta_1 \varepsilon(k) + \beta_2 \varepsilon(k-1) + \dots + \beta_{n-1} \varepsilon(k-n+2), \quad (18)$$

Using the mean value \bar{y} of the process $y(k)$, (18) is written in the form

$$y(k+1) - \bar{y} = a_1 [y(k) - \bar{y}] + a_1 [y(k-1) - \bar{y}] + \dots + a_n [y(k-n+1) - \bar{y}] + \varepsilon(k+1) + \beta_1 \varepsilon(k) + \beta_2 \varepsilon(k-1) + \dots + \beta_{n-1} \varepsilon(k-n+2), \quad (19)$$

$$[y(k+n) - \bar{y}] - a_1 [y(k) - \bar{y}] + a_1 [y(k+n-1) - \bar{y}] - \dots - a_n [y(k) - \bar{y}] = \varepsilon(k+n) + \beta_1 \varepsilon(k+n-1) + \beta_2 \varepsilon(k+n-2) + \dots + \beta_{n-1} \varepsilon(k+1), \quad (20)$$

where $a_j = \beta_j = 0, \forall j > \max\{p, q\}$.

We introduce the variables of states

$$\left. \begin{aligned} x_1(k) &= y(k) - \bar{y} \\ x_2(k) &= x_1(k+1) = y(k+1) - \bar{y} \\ x_3(k) &= x_3(k+1) = y(k+2) - \bar{y} \\ &\vdots \\ x_n(k) &= x_{n-1}(k+1) = y(k+n-1) - \bar{y} \\ x_n(k+1) &= y(k+n) - \bar{y} \end{aligned} \right\} \quad (21)$$

Then we have

$$y(k+n) - \bar{y} = x_n(k+1) = a_1 x_n(k) + a_2 x_{n-1}(k) + \dots + a_n x_1(k) + \varepsilon(k+n) + \beta_1 \varepsilon(k+n-1) + \beta_2 \varepsilon(k+n-2) + \dots + \beta_{n-1} \varepsilon(k+1).$$

Let us consider the following model in the space of states

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ \vdots \\ x_n(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ a_n & a_{n-1} & a_{n-2} & \dots & a_1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ \vdots \\ x_n(k) \end{bmatrix} + \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_n \end{bmatrix} \varepsilon(k+1), \quad (22)$$

$$y(k) = \bar{y} + [1 \ 0 \ \dots \ 0] \begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ \vdots \\ x_n(k+1) \end{bmatrix}. \quad (23)$$

The perturbation matrix in (22) is defined as follows

$$\begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_n \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ -a_1 & 1 & 0 & \dots & 0 \\ -a_2 & -a_1 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & 0 \\ -a_{n-1} & -a_{n-2} & \dots & -a_1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ \beta_1 \\ \beta_2 \\ \vdots \\ \beta_{n-1} \end{bmatrix}. \quad (24)$$

It is not difficult to see that

$$x_i(k) = [y(k+i-1) - \bar{y}] - \sum_{j=2}^i g_{i-j} \varepsilon(k+i-j+1), \quad i \leq n, \quad (25)$$

$$x_n(k) = [y(k+n-1) - \bar{y}] - g_1 \varepsilon(k+n-1) - \dots - g_{n-1} \varepsilon(k+1). \quad (26)$$

$$x_n(k) = a_n x_1(k-1) + a_{n-1} x_2(k-1) + \dots + a_1 x_n(k-1) + g_n \varepsilon(k). \quad (27)$$

From representations (25), (26) and (27) we obtain

$$\begin{aligned} & [y(k+n-1) - \bar{y}] - g_1 \varepsilon(k+n-1) - \dots - g_{n-1} \varepsilon(k+1) = \\ & = a_n [y(k-1) + \bar{y}] + a_{n-1} \{ [y(k) - \bar{y}] - g_1 \varepsilon(k) \} + \\ & + a_{n-2} \{ [y(k+1) - \bar{y}] - g_1 \varepsilon(k+1) - g_2 \varepsilon(k) \} + \dots + \\ & + a_1 \{ [y(k+n-2) - \bar{y}] - g_1 \varepsilon(k+n-2) - \dots - g_{n-1} \varepsilon(k) \} + g_n \varepsilon(k), \\ & [y(k+n-1) - \bar{y}] - a_1 [y(k+n-2) - \bar{y}] - \dots - a_n [y(k-1) - \bar{y}] = \\ & = -a_{n-1} g_1 \varepsilon(k) + a_{n-2} [g_1 \varepsilon(k+1) + g_2 \varepsilon(k)] - \dots - \\ & + a_1 [g_1 \varepsilon(k+n-2) + \dots + g_{n-1} \varepsilon(k)] + \\ & + \varepsilon(k+n-1) + \beta_1 \varepsilon(k+n-2) + \beta_2 \varepsilon(k+n-3) + \dots + \beta_{n-1} \varepsilon(k), \\ & \begin{cases} g_1 = 1 \\ g_2 - a_1 g_1 = \beta_1 \\ \vdots \\ g_n - a_1 g_{n-1} - \dots - a_{n-1} g_1 = \beta_{n-1} \end{cases}, \\ & \begin{bmatrix} 1 \\ \beta_1 \\ \beta_2 \\ \vdots \\ \beta_{n-1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ -a_1 & 1 & 0 & \dots & 0 \\ -a_2 & -a_1 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -a_{n-1} & -a_{n-2} & \dots & -a_1 & 1 \end{bmatrix} \begin{bmatrix} g_1 \\ g_2 \\ g_3 \\ \vdots \\ g_n \end{bmatrix}. \end{aligned}$$

The last equality coincides with (24) and model (20) is written in the form

$$x(k+1) = Ax(k) + Cw(k+1),$$

$$z(k) = Bx(k).$$

where

$$x(k) = \begin{bmatrix} x_1(k) \\ x_2(k) \\ \vdots \\ x_n(k) \end{bmatrix}, \quad A = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ a_n & a_{n-1} & a_{n-2} & \dots & a_1 \end{bmatrix}, \quad w(k+1) = [\varepsilon(k+1)],$$

$$C = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ -a_1 & 1 & 0 & \cdots & 0 \\ -a_2 & -a_1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -a_{n-1} & -a_{n-2} & \cdots & -a_1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ \beta_1 \\ \beta_2 \\ \vdots \\ \beta_{n-1} \end{bmatrix}, \quad z(k) = y(k) - \bar{y},$$

$$B = [1 \ 0 \ \cdots \ 0].$$

Corollary. Let us consider the following auto regression model [7]:

$$p(k) = a_0 + a_1 p(k-1) + a_2 p(k-2),$$

where $p(k)$ is the consumer prices index at a moment of time k , a_0 , a_1 , a_2 are the numerical coefficients which are calculated by statistical data on $p(k)$. For this model we have

$$x_1(k) = p(k),$$

$$x_2(k) = x_1(k-1) = p(k-1),$$

$$\begin{bmatrix} p(k+1) \\ p(k) \end{bmatrix} = \begin{bmatrix} a'_0 \\ 0 \end{bmatrix} + \begin{bmatrix} a_1 & a_2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} p(k) \\ p(k-1) \end{bmatrix} + \begin{bmatrix} \gamma_1 \\ 0 \end{bmatrix} u(k) + \begin{bmatrix} \beta_1 \\ 0 \end{bmatrix} \varepsilon(k+1),$$

$$z(k) + p(k) = [1 \ 0] \begin{bmatrix} p(k) \\ p(k-1) \end{bmatrix}.$$

We introduce the following notations

$$x(k) = \begin{bmatrix} p(k) \\ p(k-1) \end{bmatrix}, \quad a = \begin{bmatrix} a'_0 \\ 0 \end{bmatrix}, \quad \Phi = \begin{bmatrix} a_1 & a_2 \\ 1 & 0 \end{bmatrix}, \quad \Psi = \begin{bmatrix} \gamma_1 \\ 0 \end{bmatrix}, \quad G = \begin{bmatrix} \beta_1 \\ 0 \end{bmatrix},$$

$$u(k) = [u(k)], \quad w(k+1) = [\varepsilon(k+1)], \quad H = [1 \ 0].$$

Then the equations of states and dimensions take respectively the form

$$x(k+1) = \Phi x(k) + \Psi u(k) + a + Gw(k+1),$$

$$z(k) = Hx(k).$$

Example. Let us consider data on the consumer prices index in Georgia in 2002-2013 (see [5]): Table

Year	2002	2003	2004	2005	2006	2007
Index	104.4	105.0	5106.3	108.4	106.9	110.7
Year	2008	2009	2010	2011	2012	2013
Index	108.3	111.3	101.6	111.4	101.3	99.1

The problem consists in the estimate of the coefficients a_0 , a_1 , a_2 of model (1). We have the following system of equations

$$\begin{cases} a_0 + 105.6a_1 + 104.6a_2 = 104.4 \\ a_0 + 104.4a_1 + 105.6a_2 = 105.5 \\ a_0 + 105.5a_1 + 104.4a_2 = 106.3 \\ a_0 + 106.3a_1 + 105.5a_2 = 108.4 \\ a_0 + 108.4a_1 + 106.3a_2 = 106.9 \\ a_0 + 106.9a_1 + 108.4a_2 = 110.7 \\ a_0 + 110.7a_1 + 106.9a_2 = 108.8 \\ a_0 + 108.8a_1 + 110.7a_2 = 111.3 \\ a_0 + 111.3a_1 + 108.8a_2 = 101.6 \\ a_0 + 101.6a_1 + 111.3a_2 = 111.4 \\ a_0 + 111.4a_1 + 101.6a_2 = 101.3 \\ a_0 + 101.3a_1 + 111.4a_2 = 99.10 \end{cases}$$

We have to find the maximum point of the function

$$\begin{aligned} g(a_0, a_1, a_2) = & (a_0 + 105.6a_1 + 104.6a_2 - 104.4)^2 + \\ & + (a_0 + 104.4a_1 + 105.6a_2 - 105.5)^2 + (a_0 + 105.5a_1 + 104.4a_2 - 106.3)^2 + \\ & + (a_0 + 106.3a_1 + 105.5a_2 - 108.4)^2 + (a_0 + 108.4a_1 + 106.3a_2 - 106.9)^2 + \\ & + (a_0 + 106.9a_1 + 108.4a_2 - 110.7)^2 + (a_0 + 110.7a_1 + 106.9a_2 - 108.8)^2 + \\ & + (a_0 + 108.8a_1 + 110.7a_2 - 111.3)^2 + (a_0 + 111.3a_1 + 108.8a_2 - 101.6)^2 + \\ & + (a_0 + 101.6a_1 + 111.3a_2 - 111.4)^2 + (a_0 + 111.4a_1 + 101.6a_2 - 101.3)^2 + \\ & + (a_0 + 101.3a_1 + 111.4a_2 - 99.10)^2. \end{aligned}$$

Using the least square method, we obtain the following system of algebraic equations

$$\begin{cases} 12a_0 + 1282.2a_1 + 1285.5a_2 = 1275.7 \\ 1282.2a_0 + 137132.66a_1 + 137308.7a_2 = 136322.87 \\ 1285.5a_0 + 125585.57a_1 + 137812.12a_2 = 136694.66 \end{cases}$$

The solution of this system is

$$\begin{cases} a_0 = 141.3122843 \\ a_1 = -0.0058061 \\ a_2 = -0.3209668 \end{cases}$$

Therefore an auto regression model of the consumer prices index in Georgia has the form

$$p(k) = 141.3122843 - 0.0058061p(k-1) - 0.3209668p(k-2) + u(k) + \varepsilon(k).$$

Neglecting the control $u(k)$ and a random factor $\varepsilon(k)$ ($u(k) = \varepsilon(k) = 0$), we obtain the expected value of the consumer prices index of Georgia in 2014 as compared with that in the preceding year:

$$p(2014) = 141.3122843 - 0.0058061 \cdot 99,1 - 0.3209668 \cdot 101.3 \approx 108\%.$$

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